96 2 資格考

輸送現象

1. (15 points) Suppose that for a fluid flowing in a tube, the temperature distribution is $T - T_0 = \frac{\mu v_{\max}^2}{4\kappa}\left[1-\left(\frac{r}{R}\right)^4\right]$ and the velocity distribution is $v = v_{\max}\left[1-\left(\frac{r}{R}\right)^2\right]$. Find the cross-sectional average temperature T_{av} and the cup-mixing (or bulk) temperature T_b.

2. (15 points) Transpiration cooling in a planar system. Two large flat porous horizontal plates are separated by a relatively small distance L. The upper plate at y = L is at temperature T_L, and the lower one at y = 0 is to be maintained at a lower temperature T_0. To reduce the amount of heat that must be removed from the lower plate, an ideal gas at T_0 is blown upward through both plates at a steady rate. Develop an expression for the temperature distribution and the amount of heat q_0 that must be removed from the cold plate per unit area as a function of the fluid properties and gas flow rate. Use the abbreviation $\phi = \frac{\rho C_P v_y L}{k}$.

3. A very long naphthalene cylindrical with initial radius R is hanging in the air. Suppose the saturation pressure of naphthalene in the air is P_n.
   (a) (10 points) Find the concentration distribution of naphthalene in the air.
   (b) (10 points) Find the time required for the diameter of the cylinder to reduced to $\frac{R}{2}$.

4. (15 points) Explain the following terms.
   (a) Continuum theory
   (b) Reynolds number
   (c) Prandlt mixing length
   (d) Stokes flow
   (e) Eddy viscosity

5. (15 points) A solid cylinder of diameter a_1 is housed inside a hollow cylinder of diameter a_2. These two are kept concentric when the inner cylinder rotates at an angular velocity Ω. Assuming that the fluid is non-Newtonian, incompressible, isothermal, and at steady state, calculate the velocity distribution and the torque for the cylinder to rotate.

6. (20 points) For a non-Newtonian, incompressible, isothermal, and steady state fluid flowing past a semi-infinite flat plate with an uniform approaching velocity U_∞, a laminar boundary layer forms over the flat plate. Simplify the Navier-Stokes equations and use the von Kármán integral method to estimate the drag coefficient and boundary thickness by assuming that the velocity distribution inside the boundary layer can be described by $v_x = a \sin by$.

Cartesian Coordinate

$$\rho\left(\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right)+\rho g_z$$

Cylindrical Coordinate

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+\frac{u_\theta^2}{r}+u_z\frac{\partial u_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left(\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(ru_r)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right)+\rho g_r$$

$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r u_\theta}{r}+u_z\frac{\partial u_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left(\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(ru_\theta)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right)+\rho g_\theta$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}+u_z\frac{\partial u_r}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_z}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right)+\rho g_z$$

Spherical Coordinate

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial u_r}{\partial \phi}-\frac{u_\phi^2+u_\theta^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left(\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left(r^2u_r\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u_r}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_r}{\partial \phi^2}\right)+\rho g_r$$

$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial u_\theta}{\partial \phi}+\frac{u_r u_\theta}{r}-\frac{u_\phi^2\cot\theta}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial u_\theta}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left[\frac{1}{\sin\theta}\frac{\partial(u_\theta\sin\theta)}{\partial \theta}\right]+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\theta}{\partial \phi^2}\right.$$
$$\left.+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{2}{r^2}\frac{\cos\theta}{\sin^2\theta}\frac{\partial u_\phi}{\partial \phi}\right)+\rho g_\theta$$

$$\rho\left(\frac{\partial u_\phi}{\partial t}+u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\phi}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}+\frac{u_r u_\phi}{r}+\frac{u_\theta u_\phi\cot\theta}{r}\right)=-\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}\mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial u_\phi}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left[\frac{1}{\sin\theta}\frac{\partial(u_\phi\sin\theta)}{\partial \theta}\right]+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\phi}{\partial \phi^2}\right.$$
$$\left.+\frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial \theta}+\frac{2}{r^2}\frac{\cos\theta}{\sin^2\theta}\frac{\partial u_\phi}{\partial \phi}\right)+\rho g_r$$