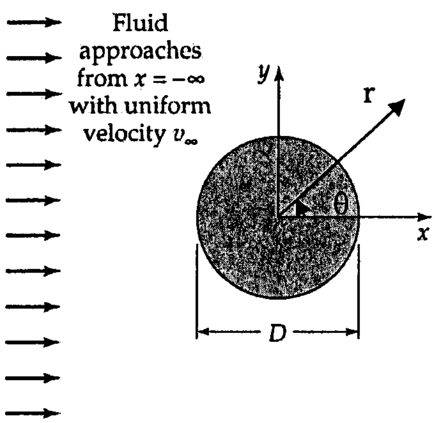
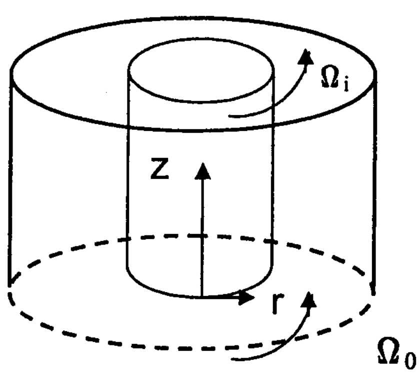
Qualify Examine (Fall, 2006)

Transport Phenomena **Dec. 26, 2006**

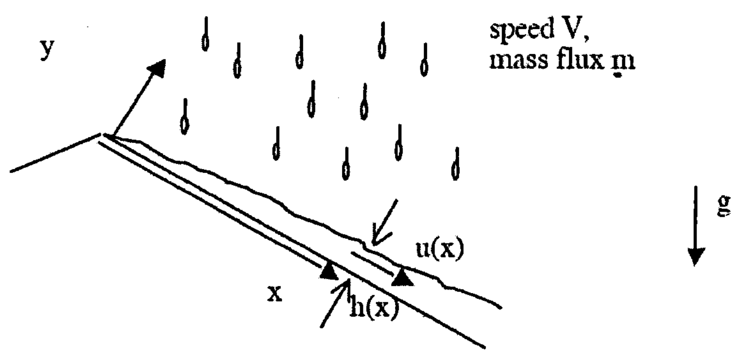
1. Consider the incompressible and inviscid fluid in potential flow around a cylinder as shown in the following figure. Find the velocity components v_r and v_θ. (15%)



2. The surface between two long coaxial cylinders is filled with an incompressible Newtonian fluid as shown in the following figure. The radii of the inner and outer wetted surfaces are kR and R, respectively. The angular velocities of rotation of the inner and outer cylinders are Ω_i and Ω_0. Determine the velocity profile between the two cylinders. (15%)



3. A violent rainstorm hits a roof inclined at an angle θ from the horizontal as shown in the following figure. The rain pours down at a mass flow rate m per unit horizontal area, each drop falls at a velocity V. Soon a steady-state water (density ρ) layer is established, while raindrops splash violently on the top part of the layer. The angle of the water surface relative to the roof is small (dh/dx<<1), and friction between roof and water may be neglected. The roof inclination θ is neither small nor large. Derive a solution h(x) in this case (20%).



$$[\partial\rho/\partial t + (\nabla \cdot \rho \mathbf{v}) = 0]$$

Cartesian coordinates (x, y, z):

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad \text{(B.4-1)}$$

Cylindrical coordinates (r, θ, z):

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad \text{(B.4-2)}$$

Spherical coordinates (r, θ, φ):

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0 \qquad \text{(B.4-3)}$$

$$[\rho Dv/Dt = -\nabla p + \mu \nabla^2 v + \rho g]$$

Cartesian coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x \quad \text{(B.6-1)}$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y \quad \text{(B.6-2)}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad \text{(B.6-3)}$$

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r \quad \text{(B.6-4)}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta \quad \text{(B.6-5)}$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad \text{(B.6-6)}$$

Spherical coordinates (r, θ, φ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right] + \rho g_r \quad \text{(B.6-7)}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right] + \rho g_\theta \quad \text{(B.6-8)}$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right] + \rho g_\phi \quad \text{(B.6-9)}$$

4. A composite wall with cross-sectional area of 10 m^2 insulates a warm room (25°C) from outside cold air (5°C). The convective heat transfer coefficients for the warm air (inside) and cold air (outside) are 10 $W/m^2K$and 20 W/m^2K, respectively. The composite wall is made of 3 layers of material which is (starting from the cold air side) concrete (thickness 0.1 m，k = 1.00 W/m.K)、fiber glass (0.02 m，k = 0.05 W/m.K) and wood (0.05 m，k = 0.25 W/m.K). Find the power (watt) required for the heater inside the room to keep room temperature at 25°C. (25 %)

5. A naphthalene ball with initial radius R is hanging in stationary air by a very thin wire. Find the time required for the ball radius to reduce to 0.5 R. (25%) [Hint: assume pseudo-steady-state first and find the rate of sublimation]

Quality Exam : Transport Phenomena June 4, 2007

Close-Book Exam

1. (25%) A semi-infinite body of liquid with constant density ρ and viscosity μ is bounded below by a horizontal surface (the *xz*-plane, length *L* and width *W*). Initially the fluid and the solid are at rest. Then at time $t = 0$, the solid surface is set in motion in the positive *x* direction with velocity v_0.
   a. Make suitable assumptions and write down the resulting simplified equations of continuity and motion. State suitable boundary conditions.
   b. Find the velocity v_x as a function of *y* and *t*.

2. (25%) The decomposition of A_n to form *A* occurs on a catalytic surface according to the reaction $A_n \rightarrow n\,A$. This reaction is so rapid that diffusion through the stagnant film over the catalyst controls the rate of the decomposition.

   The catalyst is a flat plate of length *L* and width *W*. Develop an expression for the steady-state decomposition rate in terms of the fluid properties, the mass diffusivity ***D***, the molar fraction of A_n in the bulk of the fluid phase, $y_{An,\delta}$, and the thickness of the stagnant film δ. Assume this is a gas phase reaction operated at constant *T* and *P*.

3. (15%) A solid sphere of radius R is rotating slowly at a constant angular velocity W in a large body of quiescent fluid as shown in Fig. A. Develop expression for velocity distributions in the fluid and for the torque T_z required to maintain the motion. Assume creeping flow is valid. The ϕ-component of the Navier stokes equation is given as:

$$\rho\left(\frac{\partial v_\varphi}{\partial t} + v_r\frac{\partial v_\varphi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\varphi}{\partial \theta} + \frac{v_\varphi}{\sin\theta}\frac{\partial v_\varphi}{\partial \varphi} + \frac{v_\varphi v_r + v_\theta v_\varphi \cot\theta}{r}\right) = \rho g_\varphi - \frac{1}{r\sin\theta}\frac{\partial p}{\partial \varphi}$$

$$+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\varphi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\varphi \sin\theta i\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\varphi}{\partial \varphi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \varphi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \varphi}\right]$$

[Hint: $\vec{v} = v_\varphi(r,\theta)\vec{e}_\varphi$ only].

4. (15%) The von-Karman momentum balance is given as

$$-\tau_w = \mu\left(\mu\frac{\partial v_x}{\partial y}\right)_{y=0} = \frac{d}{dx}\left(\int_0^\infty \rho v_x(v_\infty - v_x)dy\right) + \frac{dv_\infty}{dx}\int_0^\infty \rho(v_\infty - v_x)dy .$$ Assume

for laminar boundary layer flow over flat plate, velocity inside the boundary layer can be approximated by $\frac{v_x}{v_\infty} = a + b\sin(c\eta)$ where v_∞ is the constant free stream velocity, $\eta = y/\delta$ and δ is the boundary layer thickness which depends on x. (a) Determine constant a, b and c by using appropriate boundary conditions [Hint: one of the boundary boundary condition is at $y = \delta$, $dv_x/dy = 0$]; (b) Find δ/x and C_D as function of Re_x.

5. (20%) A spherical catalyst particle has a radius R and a thermal conductivity k. Due to chemical reaction, heat is generated in the porous catalyst particle at a rate of S_c W/m^3. Heat is lost to the outside gas stream with a constant temperature T_g by convective heat transfer with heat transfer coefficient h. (a) Set up the energy differential equation; (b) Find the temperature profile; (c) What is the limiting form of (b) as $h \to \infty$?; (d) Find the maximum temperature in the particle.

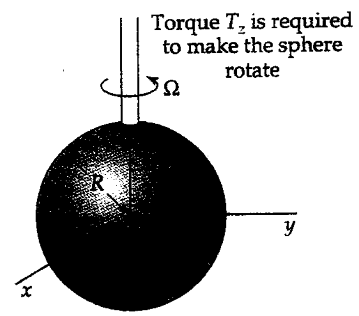


Figure A