Qualification exam (transport phenomena)

Problem 0.1. Draw diagrams to show the force direction of a shear stress and the surface element on which the shear stress is acting on, (a) $\tau_{r\theta}$ in a cylindrical coordinate; (5%) (b) $\tau_{\theta\phi}$ in a spherical coordinate. (5%) You must follow the convention of defining the shear stress in transport phenomena textbooks.

Problem 0.2. We are discussing problems of the mass-transfer boundary layer in a vacuum chamber. The issue concerns the variation of boundary layer thickness on a plate in the vacuum chamber. The question is directed on "The boundary layer thickness shall increase or decrease when the chamber pressure decreases". What is your judgement on this question?(5%) A related question is that "the boundary layer problem is more important or less important in terms of mass transfer when the chamber pressure decreases". What is your opinion?(5%)

Problem 0.3. Uranium hexafluoride UF_6 is an intermediate chemical in purification of uranium since it is relatively easy to vaporize. One method to produce UF_6 vapor is to expose the low-grade uranium ore to HF vapor at high temperatures. Suppose the uranium ore is spherical in shape and the molecular diffusion to the pellet surface is the rate controlling step. And the reaction that occurs on the pellet surface is

U(s) + 6 HF(g) $\rightarrow$ UF_6(g) + 3 H_2(g)

(s) : solid, (g): gas

Formulate and solve the steady state mass transfer problem under the assumption that the shrinkage of spherical pellet is negligible. Derive the production rate of UF_6 in mole per unit time for one pellet, assuming that the surface molar fraction of UF_6 is 1/4. Please use the following notation, (30%)

$y_{\mathrm{UF_6}}$: molar fraction of UF_6
C : total molar concentration in the gas phase
D_{AB} : the diffusivity of UF_6 in the gas phase
R : radius of the pellet
$N_{\mathrm{UF_6}}$, N_{H_2}, N_{HF} : the molar fluxes of UF_6 , H_2, HF relative to a fixed coordinate.

Problem 0.4. Two immiscible liquids, A and B, are flowing in laminar flow between two parallel plates. Is it possible that the velocity profiles would be of the following form (Figure 4)? Explain. (10%)

Problem 0.5. Consider an incompressible, Newtonial, planar flow in two dimensions, $\vec{v} = u\,\vec{i} + v\,\vec{j}$, where u and v depends only on (x, y).

If $u(x, y)$ is known, how would you calculate $v(x, y)$? (5 %)

How would one calculate the total pressure, $P(x, y)$? (5 %)

A sream function, $\Psi(x, y)$, is defined by $u = \frac{\partial \Psi}{\partial y}$, $v = -\frac{\partial \Psi}{\partial x}$. What equation of fluid dynamics does Ψ satisfy automatically by its definition? (5 %)

Problem 0.6. Use the film theory to illustrate the Chilton and Colburn J-factor analogy for convective heat transfer. (10%)

Problem 0.7. Fins are used to increase the area available for heat transfer between metal walls and poorly conducting fluids, such as gases. Figure 7 shows a finned heat exchanger with an uniform cross sectional area $A(z)$ and a perimeter $P(z)$. The wall temperature of the fin is T_w and the ambient temperature is T_∞. Temperature is assumed to be a function of z only, and heat lost from the end of the fin is negligible. The heat fux at the surface can be given by $q_z = h(T - T_\infty)$, where h is constant. The thermal conductivity of solid is k. Derive a governing equation for the finned heat exchanger at unsteady state, and give the boundary and initial conditions. (15 %)

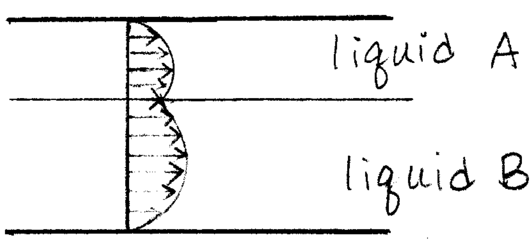


Figure 4

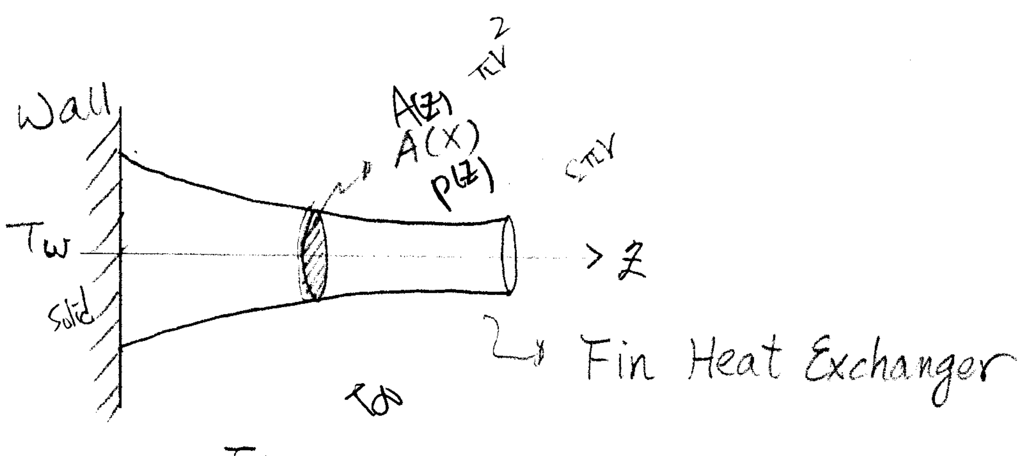


Figure 7

Qualifying Examine

Transport Phenomena

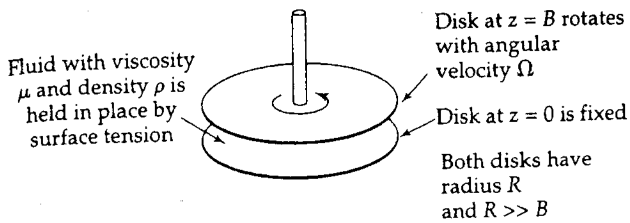
April 11, 2005

1. (20%) A fluid, whose viscosity μ is to be measured, is placed in the gap of thickness B between the two disks of radius R. One measures the torque T_z required to turn the upper disk at an angular velocity Ω. Develop the formula for deducing the viscosity from the measurements. Assume creeping flow.

   a. Make suitable assumptions and write down the resulting simplified equations of continuity and motion. State suitable boundary conditions.

   b. Assume $v_\theta = r f(z)$. Solve v_θ.

   c. Find the expression of T_z in B, μ, Ω, R.



2. (10%) For a rotational flow with $v_x = -by, v_y = bx, v_z = 0$, find $\nabla \times \underline{v}$ and the angular velocity of the flow.

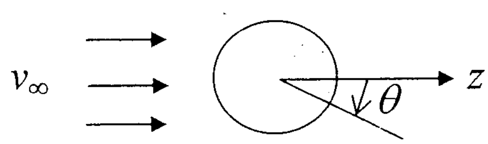
3. (20%)

(a) Use the attached table to set up the differential equation and boundary conditions for the stream function of a steady creeping flow of a Newtonian fluid around a stationary sphere of radius R. The fluid approaches the sphere in the positive z direction with a v_∞ velocity, as shown in the following figure.

(b) According to one of the boundary conditions, we may suggest that the stream function $\Psi(r, \theta)$ is of the form

$$\Psi = f(r) \sin^2\theta$$

Solve the stream function and the velocity distribution (v_r and v_θ) of the flow.



Cylindrical coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Table Equations for the Stream Function

Type of motion	Coordinate system	Velocity components	Differential equations for ψ which are equivalent to the Navier–Stokes equation[b]	Expressions for operators
Two-dimensional (planar)	Rectangular with $v_z = 0$ and no z-dependence	$v_x = -\frac{\partial \psi}{\partial y}$ $v_y = +\frac{\partial \psi}{\partial x}$	$\frac{\partial}{\partial t}(\nabla^2\psi) + \frac{\partial(\psi, \nabla^2\psi)}{\partial(x, y)} = \nu\nabla^4\psi$ (A)	$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ $\nabla^4\psi \equiv \nabla^2(\nabla^2\psi)$ $\equiv \left(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}\right)\psi$
	Cylindrical with $v_z = 0$ and no z-dependence	$v_r = -\frac{1}{r}\frac{\partial \psi}{\partial \theta}$ $v_\theta = +\frac{\partial \psi}{\partial r}$	$\frac{\partial}{\partial t}(\nabla^2\psi) + \frac{1}{r}\frac{\partial(\psi, \nabla^2\psi)}{\partial(r, \theta)} = \nu\nabla^4\psi$ (B)	$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$
Axisymmetrical	Cylindrical with $v_\theta = 0$ and no θ-dependence	$v_z = -\frac{1}{r}\frac{\partial \psi}{\partial r}$ $v_r = +\frac{1}{r}\frac{\partial \psi}{\partial z}$	$\frac{\partial}{\partial t}(E^2\psi) - \frac{1}{r}\frac{\partial(\psi, E^2\psi)}{\partial(r, z)} - \frac{2}{r^2}\frac{\partial \psi}{\partial z}E^2\psi = \nu E^4\psi$ (C)	$E^2 \equiv \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$ $E^4\psi \equiv E^2(E^2\psi)$
	Spherical with $v_\phi = 0$ and no ϕ-dependence	$v_r = -\frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta}$ $v_\theta = +\frac{1}{r\sin\theta}\frac{\partial \psi}{\partial r}$	$\frac{\partial}{\partial t}(E^2\psi) + \frac{1}{r^2\sin\theta}\frac{\partial(\psi, E^2\psi)}{\partial(r, \theta)}$ $-\frac{2E^2\psi}{r^2\sin^2\theta}\left(\frac{\partial \psi}{\partial r}\cos\theta - \frac{1}{r}\frac{\partial \psi}{\partial \theta}\sin\theta\right) = \nu E^4\psi$ (D).	$E^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\right)$

4. A viscous fluid with temperature-independent physical properties is in fully developed laminar flow between two flat surfaces placed a distance 2B apart. The velocity distribution can be expressed as $v_z = v_{max}\left(1-\frac{x^2}{B^2}\right)$. Assume for $z \le 0$ the fluid temperature is uniform at $T = T_1$. For $z > 0$ heat is added at a constant, uniform flux q_o at both walls. (a) Start from the energy equation $\rho C_p \frac{DT}{Dt} = k\nabla^2 T + \mu\Phi$, write down the differential equation for T(x, z) by neglecting the viscous dissipation term and the axial heat conduction term. Also write down the corresponding boundary conditions. (5%) (b) Put the differential equation for T(x, z) in (a) into the following dimensionless form $(1-\sigma^2)\frac{\partial\theta}{\partial\xi} = \frac{\partial^2\theta}{\partial\sigma^2}$ with $\theta = \frac{T-T_1}{q_0 B/k}$, $\sigma = x/B$; what is ξ? Write down the corresponding boundary conditions in dimensionless form. (5%)
(c) Assume the asymptotic solution for large ξ can be expressed as $\theta(\sigma, \xi) = C_0\xi + \Phi(\sigma)$, find C_0 and $\Phi(\sigma)$. Note that you have to replace boundary condition: at $z = 0$, $T = T_1$ by an energy balance over the flat surface:

$$z \times 1 \times q_0 = \int_0^B \rho C_p v_z (T - T_1)dx \quad (10\%)$$

(d) Find the Nusselt number $Nu = h(2B)/k$, where h is defined by $q_o = h(T_w - T_b)$, T_w and T_b are the wall temperature and the cup-mixing temperature, respectively. (10%)

5. A tenant was found dead in his rented house. A police (John) was called to investigate the case. Firstly John has to decide when the man died. John noticed a cup of tea on the table. Apparently before the man died, he was having a cup of tea. Because water evaporates very slowly so there is a mark on the cup that tells how much water level has dropped since the man drop dead. John take a measurement and found the distance between the surface of water to the top of the cup to be L, and the distance from the mark to the top of the cup to be H (L > H). John measured the room temperature (T_a), the relative humility in the room (so the partial pressure of water P_w). From Perry's Handbook John obtained the saturated water pressure at T_a to be P_S. From the above data, John was able to estimate how long the man has died. Explain how John did it. (20%)